

**34.9** A boiler provides  $10,000 \frac{lb}{hr}$  of  $900psia$  saturated steam to a turbine which produces  $800KW$ . Steam exits the turbine to a condenser from which it leaves as a  $14.7psia$  liquid with  $10^\circ F$  of sub-cooling. The liquid is pumped back to the boiler to continue the cycle. Neglecting pump work, what is the efficiency of the cycle?

- A. 22%
- B. 28%
- C. 37%
- D. 44%

Use the equation for the efficiency of a **Rankine Cycle**. The reference handbook states the efficiency in terms of the enthalpies at each state. Another useful and more general presentation of the efficiency is useful here.

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}}$$

The net work is formally the difference between the work produced by the turbine and the work required to drive the pump. However, in this case, and often with similar Rankine cycle problems, the pump work is considered to be negligible. Convert the turbine output to units of  $\frac{Btu}{hr}$ .

$$\dot{W}_{net} = \dot{W}_{turbine} - \dot{W}_{pump}$$

$$\dot{W}_{pump} \approx 0$$

$$\dot{W}_{net} = \dot{W}_{turbine} = 800KW \left( 3412 \frac{Btu}{hr \cdot KW} \right) = 2.73 \times 10^6 \frac{Btu}{hr}$$

The heat into the cycle,  $\dot{Q}_{in}$ , is entirely a function of the boiler. Since the mass flow rate is known, represent the heat in using  $\dot{Q} = m\Delta h$ . In alignment with the Rankine Cycle diagram in the reference handbook, consider the turbine entering condition as State 3, the turbine exit as State 4, the condenser exit as State 1, and the pump exit as State 2.

$$\dot{Q}_{in} = \dot{m} (h_3 - h_2)$$

Use the properties of **Saturated Water and Steam** to obtain the enthalpy at State 3.

$$P_3 = 900psia \text{ (saturated)}$$

$$h_3 = h_g \approx 1150 \frac{Btu}{lb}$$

For State 2, recall that the pump work is negligible. Conclude that  $h_2 \approx h_1$ .

$$\dot{W}_{pump} = \dot{m} (h_2 - h_1) \approx 0$$

$$h_2 \approx h_1$$

For State 1, the enthalpy will be a one-for-one reduction from the enthalpy for saturated liquid for each degree of subcooling, as shown formally below. Refer to the properties of **Saturated Water and Steam** to obtain  $h_{sat} = h_{f@P_1}$ .

$$P_1 = 14.7 \text{ psia}$$

$$T_1 = 212^\circ F - 10^\circ F = 202^\circ F$$

$$\Delta h = c_p (\Delta T)$$

$$h_{sat} - h_1 = c_p (T_{sat} - T_1)$$

$$h_1 = h_{sat} - c_p (T_{sat} - T_1) = 180 \frac{\text{Btu}}{\text{lb}} - \left( 1 \frac{\text{Btu}}{\text{lb} \cdot ^\circ F} \right) (212^\circ F - 202^\circ F) = 170 \frac{\text{Btu}}{\text{lb}}$$

Calculate  $\dot{Q}_{in}$ .

$$h_2 \approx h_1 \approx 170 \frac{\text{Btu}}{\text{lb}}$$

$$\dot{Q}_{in} = \dot{m} (h_3 - h_2) = \left( 10,000 \frac{\text{lb}}{\text{hr}} \right) \left( 1150 \frac{\text{Btu}}{\text{lb}} - 170 \frac{\text{Btu}}{\text{lb}} \right) = 9.8 \times 10^6 \frac{\text{Btu}}{\text{hr}}$$

Determine the efficiency of the cycle.

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{2.73 \times 10^6 \frac{\text{Btu}}{\text{hr}}}{9.8 \times 10^6 \frac{\text{Btu}}{\text{hr}}} = 27.9\%$$

**Answer B**